

## Towards a Coherent View of Brain Connectivity

Thomas F. Collura, PhD, PE

**ABSTRACT.** *Background.* The electroencephalogram provides a myriad of opportunities to detect and assess brain function and brain connectivity.

*Method.* This article describes the relationship between local and non-local brain activation and synchrony, and discusses the use of appropriate connectivity measures to study and train functional brain connectivity. Specific connectivity measures are described including coherence, phase, synchrony, correlation, and comodulation. The measures are contrasted and compared in terms of their ability to detect particular aspects of connectivity and their usefulness for neurofeedback training.

*Results.* Connectivity metrics for example EEG data are calculated and shown graphically, to illustrate relevant principles.

*Conclusion.* It is possible to assess brain connectivity and integrated function for both assessment and training, through the use of appropriate metrics and display methods.

**KEYWORDS.** Brain connectivity, coherence, EEG, phase QEEG, quantitative electroencephalography, spectral correlation, synchrony

The electroencephalogram (EEG) is a uniquely powerful and revealing indicator of brain electrical function and one of the best methods available for assessing and monitoring neural activity in real time. Measurable scalp EEG is produced by the summation, through volume conduction, of postsynaptic potentials of the pyramidal cells within the cerebral cortex (Burgess & Collura, 1992). When cells polarize (or depolarize) in unison, the resulting potentials are added in the conducting media, leading to external fields that can be measured. This phenomenon is so pronounced that a mere 1% of cortical cells in a 1 cm<sup>2</sup> area of cortex, when acting in synchrony, are sufficient to account for more than 96% of the EEG signal (Shaw, 2003). In other words, the existence of an EEG potential implies some degree of local synchrony within a population of cells lying beneath the affected sensor. By an extension of this logic, if a mere 1% of cortical cells are coordinated in some

---

Thomas F. Collura is affiliated with BrainMaster Technologies, Inc., Oakwood Village, Ohio.  
Address correspondence to: Thomas F. Collura, PhD, PE, BrainMaster Technologies, Inc., 24490 Broadway Avenue, Suite #2, Oakwood Village, OH 44146 (E-mail: tomc1@brainm.com).

way with 1% of the cells in some other location, then 96% of the connectivity might be accounted for in the EEG. The question is, how do we define this connectivity and how do we measure it?

The brain comprises cortical centers, connections between cortical centers, and connections between cortical centers and subcortical structures (most notably the thalamus). Cortical centers are neighboring cells that act in a synchronize manner measured as an EEG wave from a single electrode sensor. The cortical centers of short-range connections between close electrode locations and long-range connections between distant electrode locations have synchrony or coordinated electrical activity. This relationship of coordinated electrical activity between EEG signals can be measured with mathematical calculations or connectivity measures. The connectivity measures reveal important differences between short-range and long-range cortical centers and are fundamentally different from the cortical center activity from a single electrode. Connectivity measures extend our existing knowledge to incorporate increasing distances, thus reflecting whole brain function as extensions and generalizations of the concepts implicit in localized brain function.

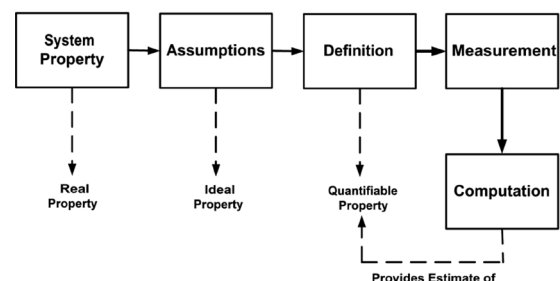
Connectivity can measure the similarity between channels in one or both of two important contexts, postprocessed and real time. In the postprocessed context, the quantitative EEG (QEEG) is examined after the entire QEEG is acquired. Fast-Fourier Transformation (FFT) and other transform-based methods are sufficient and can provide a level of precision and understandability that is of value in normative applications. However, FFT-based methods have slower time response, owing to the need to acquire an epoch of data (on the order of 1 sec) before the estimate can be made. Tapering windows further confound this delay by emphasizing wave components in the center of the window, thus imposing a firm delay of half the epoch size, thus incurring a delay of 500 msec, which maybe detrimental to EEG biofeedback applications. In contrast, the digital filters and related methods including “complex demodulation” and “joint time-frequency analysis” provide

real-time processing while retaining generality and accuracy (Collura, 1990). The main “cost” of such approaches is the need to predefine the component band of interest (e.g., 8–12 Hz).

Connectivity is concept in which mathematical calculations can be applied. Like the concept of intelligence or temperature, we make assumptions about the measure with certain understandable limitations. For example, we never measure temperature directly. By making assumptions and using definitions, we measure some other property such as the length of a column of mercury or alcohol, the deflection of a metal strip. By recording such physical entities and interpreting them in an agreed-upon way, we arrive at a measurement that we all agree to call “temperature.” The situation is not so different in the case of brain connectivity. We actually record one or more electrical potentials that we subject to computations or an agreed-upon representation. Such computations produce an estimate of a concept, which we interpret generally as the similarity between activity in the brain, and use in the pursuit of brain connectivity assessment or training. As seen in Figure 1, any connectivity measure falls within the realm of system identification and parameter estimation. By making assumptions, we derive an ideal property, which we may seek to measure. Through appropriate definitions, measurements, and computations, we arrive at an estimate of a quantifiable property, which always puts us into an abstract realm.

There are many ways or methods to measure EEG connectivity. This is alike to assessing the similarity between any two

FIGURE 1. The relationship between system properties and measured properties.



entities, be they signals, human beings, or automobiles. Any connectivity measure that describes the relationship between two sites has potential merit and may reflect the amount of information shared or the speed of information sharing. It may be applicable in real time or in a postprocessed EEG. As long as the measure of connectivity has value in assessing or training brain function, it should be considered among candidate methods and is beyond simple amplitude training. Connectivity measures may or may not have sensitivity to various properties of the electrical signal. For example, a connectivity measure may be sensitive to the phase of the signals (or it may ignore phase) but may be sensitive to the absolute or relative amplitudes of the signals or measure quantities across frequency, time, or both. Furthermore, the data source such as raw waveforms, transformed quantities using FFT, or filtered waveforms produced by digital filters or complex demodulation will affect the connectivity measure. Although these alternatives provide options for acquiring, measuring, or training brain connectivity, none of them is considered the only method to measure connectivity. The formula for a connectivity measure is not “recipe” for implementing it in a real system, any more than being told that cheesecake contains certain ingredients is a recipe for success. Critical issues such as timing, measurement of quantities, and order of computations affect may limit the usefulness. In other words, it is relatively simple to produce a connectivity measure that seems to behave as intended, but there is a wide range of considerations that must be addressed for a useful connectivity measure to be produced. Thatcher (2007) addressed these issues well in the case of classical coherence. In sum, these characteristics provide a frame-work to describe the strengths and weaknesses of connectivity measures.

### ***SPECIFIC CONNECTIVITY MEASURES***

#### ***Heart-Rate Variability (HRV) Coherence***

A measure in the field of HRV instrumentation and literature called “coherence”

is more properly called “self-coherence.” This measure is computed using only one signal or source and is a measure of the spectral purity of the energy the signal. This should not be confused with the “Coherence” measure used in EEG or QEEG, which is calculated from two signals or electrodes.

#### ***Classical Coherence***

Classical or “pure” coherence is a measure, derived from the engineering field, designed to reveal connectivity as reflected in a consistent phase relationship between two or more signals. It is defined as the cross-spectra normalized to the product of the auto-spectra and interpreted as a generalization of the Pearson correlation coefficient to variables expressed in the complex frequency domain. It has widespread use in time-series analysis (Carter, 1987) and can be expressed mathematically as

$$COHERENCE = \frac{|H_{xy}|^2}{|H_{xx}| |H_{yy}|}$$

In this representation, the numerator is the cross-spectrum between the two signals, and the two terms in the denominator represent the auto-spectra of the individual signals. This can be defined for any frequency component band or bin. When calculated across frequencies, it produces the coherence spectrum.

Pure coherence is independent of the absolute phase separation between the signals. It is independent of the individual amplitudes of the signals, in that it is possible to have high coherence between two small signals and low coherence between two large signals. It is also possible to have high coherence between a large signal and a small signal. The spectral energy of interest can be estimated by more than one method, although the classical approach is to use the Fourier Transform. When a transform is used, certain decisions are already implicit, such as the signal sampling rate; the epoch or window used for analysis; choice of windowing factors such as Hamming, Hanning, and



FIGURE 3. Relationship between two different implementations of spectral correlation coefficient. The agreement is seen to depend on the frequency response of the overall system.

correlated manner? The calculation for comodulation looks like a standard Pearson correlation coefficient:

$$\text{COMODULATION} = \frac{\sum X_{ij} Y_{ij}}{\sqrt{\sum X_{ij}^2 \sum Y_{ij}^2}}$$

In this expression, the X and Y values represent successive measurements of the signal amplitudes across time for signals X and Y, respectively. Comodulation is measured across time, so it is necessary to define the time duration and interval of measure during the computation. A comodulation value depends on these parameters, as well as the exact conditions of the detection of the amplitude data.

characteristics of the amplifiers dominates the computation. This illustrates the importance of matching the frequency response of a system, when implementing a measure. In this particular case, the tuning of the system to match the desired values consists of reducing the high-frequency response to match that of the less responsive amplifier. It is visually evident that by introducing a falloff in frequency response that has an increasing effect at higher frequencies, the two measures could be brought into well within 1% agreement. SCC can be computed for any epoch that produces an estimate of the FFT spectral energy, which is to say that it is meaningful for a single FFT sample. Thornton (2000) found this measure to be of significant value in the assessment and training of children with learning disabilities and related disorders. To this end, he constructed a database of normative values as well as clinical procedures leading to effective training.

Phase Difference There are various methods of measuring a phase. The traditional method is to calculate the arctangent of the ratio of quadrature components derived from the FFT. These computations suffer from problems including "wraparound" and related stability issues, in that as two signals are continually sliding in phase, there comes a time when they are again in phase and the measure needs to "snap" back to zero. The conditions of this transition introduce ambiguities in the definition and use of the measure. Collura (2001) described a measure that is sensitive to phase and can be derived from complex demodulation in real time. Thatcher (2007) also recently introduced a dynamic phase measure based on complex demodulation as well as a practical method of assessing (and training) phase resetting. The phase of a particular signal is generally defined as

Comodulation

$$\text{PHASE} = \frac{1}{2} \text{Arc tan} \frac{b}{a}$$

Comodulation was described by Sterman in which b represents the "imaginary" or "out-of-phase" component and a represents the "real" or "in-phase" component of the signal. Although this definition is clear for a defined signal, estimating the phase of an actual time-series is more complex (and b

can be determined by Fourier Transform, complex demodulation, or any other applicable method). The phase separation between two signals is computed by subtracting their individual phases:

$$\begin{aligned} \text{PHASED FFERENCE} = & \text{Arc tan}\left(\frac{b_2}{a_2}\right) \\ & - \text{Arc tan}\left(\frac{b_1}{a_1}\right) \end{aligned}$$

This measure also has issues with wrap-around, as well as stability and accuracy concerns raised by the need to take a ratio of two numbers that may be very small or very different in magnitude. In particular, if two signals “slide” alongside each other, there is a discontinuity in this value resulting in a need to snap the values back into alignment. Alternate forms of phase relationships can be derived, which have different properties for specific uses. There are options for expressing the reported phase measure, including degrees or percentages, in which 100% may mean either “zero degrees out of phase” (Joffe, 1992) or, alternatively, “180 degrees out of phase” (Collura, 2001).

### Similarity

Collura (2001) described a measure of “similarity” that is alike to that of classical coherence but has several important differences. First, it is maximized when the signals are in phase, which is to say that their phase difference is zero. It is a measure of synchrony, not just phase stability. Second, it is maximized when the signals are of similar size. The general form of this measure’s sensitivity is given by:

$$\text{SIMILARITY}_{[A,B]} = \frac{2AB}{A^2 + B^2}$$

in which  $A$  and  $B$  are the complex quantities of the two signals. This reaches unity when  $A$  and  $B$  are identical in both amplitude and phase. This measure produces a value that is relevant to EEG synchrony, as a special case of EEG coherence. This measure has

been found to be of particular value in the assessment of consciousness in the application of intraoperative monitoring (Alshab, Collura, & Voltz, 2005) and may be useful for QEEG biofeedback assessment and training.

### Asymmetry

Asymmetry may also be regarded as a connectivity measure, in that it reflects relative activation between the sites of interest. Asymmetry can be measured in any manner that reveals differences in signal amplitudes. Baehr and Rosenfeld (2001) derived a particularly useful measure, which takes the form of the difference between the signal amplitudes normalized to the sum of their amplitudes. This measure takes the form

$$\text{ASYMMETRY} = \frac{A - B}{A + B}$$

In this calculation,  $A$  and  $B$  are the instantaneous values of a given estimate, typically alpha amplitude, in the two channels of interest. This measure has the benefit of being independent of the individual signal amplitudes and preferring either very large amplitude differences or lower amplitude in both signals. EEG asymmetry is of particular value in working with interhemispheric EEG, in particular that of the frontal lobes as it has been found to correlate with depression (Baehr & Rosenfeld, 2001). The measurement and training of frontal EEG asymmetry has become an important avenue in clinical neurofeedback when applied to depressed patients. It is also of potential value when used intrahemispherically, particularly front to back, in which posterior amplitudes can be trained in relation to anterior amplitudes, to seek normal relationships.

### Raw Waveform Calculation

Another highly useful method is to simply add (and/or subtract) the EEG signals as raw waveforms and process the resulting signals in a conventional manner. This method

FIGURE 4. Joint time–frequency analysis of the sum (left) and difference (right) computed between two EEG signals. Synchronous and asynchronous EEG components are clearly separable.

was applied in the Capscan system and bands is predominantly independent further developed by others (McKnight & (asynchronous). This difference is significant Fehmi, 2001; Crane & Soutar, 2000). This in regard to making a choice regarding method has the benefit of being easy to monopolar or bipolar training.

understand and interpret; and is simple to use, in that the signal recombination simply produces another signal, which may then be subjected to any of the methods used for signal processing, including transforms, digital filtering, and joint time...

#### CONNECTIVITY ASSESSMENT AND TRAINING

analysis. The dependence of the channel evaluation or assessment and training is a sum or difference on the individual signal significant challenge in that it is not generally amplitudes and phases is easy to determine clear what is ••good•• in terms of any particular connectivity measure between specific locations and/or within a component band of interest. With very few exceptions, connectivity in the brain has been described as a ••Goldilocks•• aspect (D. Kaiser, July 2004, personal communication) in that the connectivity measure is too low or too high

Figure 4 shows a time...frequency plot and only needs to be ••just right•• to ensure illustrates the difference between the spectral optimal brain functioning. For this reason, energy in the sum (left) and the difference the availability of normative data is crucial (right) of signals recorded from T3 and T4. for the proper interpretation and use of connectivity measures. Walker, Kozlowski, and Lawson (2007) found that training coherence It is immediately visually evident that nectivity measures. Walker, Kozlowski, and Lawson (2007) found that training coherence without short-term guidance can lead to band, the activity in the alpha and low beta coherence abnormalities that have clinical

TABLE 1. Definition and sensitivity of EEG connectivity measures.

Parameter	Method	Looks Primarily at	Asks	Absolute Amplitude	Relative Amplitude	Absolute Phase Separation	Stability of Phase Separation	Signal Morphology
Coherence (Carter 1987)	Metric based on FFT or complex demodulation	Phase consistency over time	Is the phase relationship stable?	NO (normalized)	NO (normalized)	NO (ignores phase relationship)	YES (by definition)	NO (looks at spectral energy only)
Phase (Goodman 1957, Walter 1961)	Metric based on FFT or complex demodulation	Phase separation	What is the phase separation?	NO (normalized)	NO (normalized)	YES (highest when signals are in phase)	YES (by monitoring changes)	YES (tracks short-term phase relationships)
Similarity/ "Synchrony" (Collura 2000)	Metric based on FFT or complex demodulation	Phase separation and amplitude match	Are the signals in phase and of similar size?	NO (normalized)	YES (high when both same size)	YES (highest when signals are in phase)	YES (if high, phase must be stable)	YES (tracks short-term phase relationship)
Spectral Correlation Coefficient (Joffe 1990)	FFT: convert to amplitude; correlate shape of spectral energy across frequency and time	Spectral amplitude match across frequency and time	Do the spectral energies look similar?	NO (normalized)	NO (normalized)	NO (ignores phases relationship)	NO (insensitive to phase)	NO (looks at spectral energy only)
Comodulation (Pearson 1896, Kaiser 1994)	FFT or complex demodulation, correlate amplitudes over time	Magnitude consistency over time	Is the magnitude relationship stable?	NO (normalized)	NO (normalized)	NO (ignores phase relationship)	NO (insensitive to phase)	NO (looks at spectral energy only)
Asymmetry (Davidson, Rosenfeld)	Metric based on difference between channel magnitudes	Magnitude difference	Are the signals of different magnitudes?	NO (normalized)	YES (as defined)	NO (ignores phase relationship)	NO (insensitive to phase)	NO (looks at signal magnitude only)
Sum Channels (combined "monopolar") (Fehmi, Crane)	Add waveforms to produce new signal	waveform locking	How do the signals look when combined?	YES (highest when both are large)	YES (smaller when either is reduced)	YES (largest when signals in phase)	YES (largest when signals in phase)	YES (combines signals directly)
Difference of Channels ("bipolar") (Many)	Subtract waveforms to produce new signal	Waveform unlocking	How do the signals look when subtracted?	YES (will increase if both signals increase)	YES (largest signals differ)	YES (highest when signals out of phase)	YES (varies with phase separation)	YES (combines signals directly)

TABLE 2. Training capabilities of EEG connectivity measures.

Parameter	Can uptrain neuronal coherence	Can downtrain neuronal coherence	Can uptrain neuronal synchrony	Can downtrain neuronal synchrony	Can train neuronal dependence	Can train neuronal independence
Coherence	YES (by definition)	YES (by definition)	NO (can be high when synchrony is low)	YES (if low, synchrony must be low)	YES (reflects amount of shared information)	YES (if reduced, phase is destabilized)
Phase	YES (if change is small, coherence is (by definition) high)	YES (if phase unstable, coherence is (by definition) low)	YES (will be high when signal are synchronized)	YES (if low, synchrony must be low)	YES (reflects speed of information sharing)	YES (if reduced phases are separated)
Similarity	YES (when high, coherence must also be high)	NO (will be low if signals are coherent but out of phase)	YES (by definition)	YES (by definition)	YES (reflects amount and speed of shared information)	YES (if reduced, phase is caused to separate)
Spectral Correlation	NO (can be high when coherence is low)	NO (can be low even if signals are coherent)	NO (can be high when synchrony is low)	YES (if low, synchrony must be low)	YES (reflects similarity of neuronal activation across frequency)	YES (if low signals have uncorrelated activation across frequency)
Comodulation	NO (can be high when coherence is low)	NO (can be low even if signals are coherent)	NO (can be high when synchrony is low)	YES (if low, synchrony must be low)	YES (reflects time-coordinated neuronal activation)	YES (if low, signals have uncorrelated activation in time)
Asymmetry	NO (ignores coherence)	NO (ignores coherence)	NO (independent of synchrony)	NO (independent of synchrony)	YES (if asymmetry manifestation of dependence)	YES (trains for asymmetric activation pattern)
Sum Channels (combined "monopolar")	YES (if high, signals must be coherent)	NO (can be low even if signals are coherent)	YES (largest when signals are identical)	YES (lowest when signals are not synchronized)	YES (reflects closeness or signal details)	YES (if reduced, phase is destabilized)
Difference of Channels ("bipolar")	NO (becomes low either when signals synchronous or both small)	NO (can be low even if signals are coherent)	NO (becomes low either when signals synchronous or both small)	YES (largest when signals are different)	NO (cannot distinguish between synchronized and low-amplitude signals)	YES (largest when signals are different)

significance. When coherence is too high, it is referred to as “hypercoherence” and when coherence is too low, it is referred to as “hypocoherence,” and either of these conditions can be adverse. For example, in language areas either hypercoherence or hypocoherence can be accompanied by dysfunctions such as stuttering, word grasping, and related disorders of speech and language. Bipolar (channel difference) techniques are known to have a general effect of decreasing coherence in any bands that are trained up. This phenomenon likely plays a large role in the experience that bipolar training between T3 and T4, for example, may lead to unpredictable and uncontrolled effects, requiring frequent interviewing and coaching of the trainee. If normative data and controlled coherence training were applied in such situations, we could anticipate improved control and predictability of training and a reduction in adverse or unpredictable outcomes.

Thornton (1999) described a successful clinical approach using SCC, applied to trainees with learning disabilities. He developed his own database of normative scores and uses this in the assessment of clients and the application of neurofeedback protocols. In this approach, he honors the need for the SCC to be within a normative range. When excessively high scores are seen, down-training is indicated, and when low scores are seen, uptraining is used. A particular challenge with this approach is the fact that the normative scores are dedicated to a particular instrument (Lexicor) and do not generalize a priori to other systems. Training is based on raw scores, so that, for example, an SCC that may be low, with a value of, say, 70, needs to be trained up to a normal value of, say, 80, but not up to an abnormal level of 90. The SCC measure depends on the specific frequency response of the amplifiers; although it is straightforward to match scores in midband components (theta, alpha, and beta), they do not match in bands in which the amplifiers differ in frequency response. The exact scores must be set into the protocol and carefully watched during the training process. Normal scores depend on the sites involved, the frequency bands studies, and the age of the trainee. The

construction of a useful clinical database is time-consuming and does not generalize to other measures. For this reason, an overt effort must be made to implement a comparable measure on other platforms and to validate it before this approach can be used on them. A project with this goal is presently underway between Thornton and the author.

Collura, Thatcher, Smith, Lambos, and Stark (2008) developed a method based on an embedded database that provides real-time  $z$  scores and a specially tailored training system. When this approach is used, measures are expressed as  $z$  scores for amplitude, asymmetry, coherence, and phase. Real-time  $z$  scores differ from conventional QEEG  $z$  scores in two important ways. First, the scores are derived from real-time complex demodulation instead of the conventional FFT and are available in real time with minimal delay. Second, the real-time  $z$  scores are computed based on within-subject short-term variations *and* across-subject variations, whereas conventional QEEGs employ only across-subjects statistics. The standard deviations for real-time  $z$  scores are higher, and the resulting  $z$  scores are typically lower, and real-time  $z$  scores tend to be more “forgiving” and are more likely to appear normal.

When  $z$  scores are used for EEG training, a variety of targeting options are available. The most obvious is to train toward the norm or average range in which the protocol is designed to guide the trainee into the normal range. There are various options available when using this approach including the number of  $z$  scores available and the type of reinforcement feedback. Training to the norm may have several benefits: First, it clearly avoids the pitfalls of training either hypercoherence or hypocoherence. Second, some protocols can “auto-select” the variables that are trained, in that they will automatically ignore normal scores and will use only the most deviant scores to produce the contingent feedback. This is of particular significance in light of the fact that connectivity measures that were normal may become abnormal in the course of training. Third, the use of  $z$  scores during training may relieve practitioners of some of the burden

of a repeated cycle of QEEG/protocol design/training/repeat QEEG (Smith 2008).

### SUMMARY

This review defined and described various brain connectivity measures for QEEG analysis and neurofeedback (NF) training. Table 1 is a summary of the definitions and sensitivities of the major EEG connectivity measures during QEEG analysis. This summary can guide the researcher or clinician as to the strengths and weakness of the measure. There is substantially more information about QEEG analysis relative to NF training. The current results suggest that training to the norm or average seems the most logical and intuitive approach to NF training of connectivity measures. Clearly, there is a need for further research on which measures of connectivity are effective, at which sites, and which frequency bands. Client variables such as age, symptoms, and diagnosis may need to be included in research on NF outcome. This summary may serve as a starting point for researchers and clinicians in the area of brain connectivity. A review of Table 2 may guide research and clinical decisions for NF training with a summary of the training capabilities of the EEG connectivity measures of Coherence, Phase, Similarity, Spectral Correlation, Comodulation, Asymmetry, Summed Channels, and Difference of Channels. This table refers to Uptraining as an increase and Downtraining as a decrease to guide the choice of connectivity measure and the use of that measure during NF.

### REFERENCES

- Alshab, A., Collura, T. F., & Voltz, M. D. (2005). Proceedings from the American Society for Anesthesia Annual Meeting, October 25, 2005: *Predicting alertness with EEG processing in patients undergoing deep brain stimulator placement for Parkinson's disease. Anesthesiology* 2005, 103: A27.
- Baehr, E., & Rosenfeld, J. P. (2001) Clinical use of an alpha asymmetry neurofeedback protocol in the treatment of mood disorders: Follow-up study one to five years post therapy. *Journal of Neurotherapy*, 4(4), 11–17.
- Burgess, R., & Collura, T. F. (1992). Polarity, localization, and field determination in electroencephalography. In E. Wyllie (Ed.), *The treatment of epilepsy: Principles and practice* (pp. 211–233). Philadelphia: Lea & Feiberger.
- Carter, G. C. (1987) Coherence and time delay estimation. *Proceedings of the Institute of Electrical and Electronics Engineering*, 75, 236–255.
- Collura, T. F. (1990). Real-time filtering for the estimation of steady-state visual evoked brain potentials. *IEEE Transactions on Biomedical Engineering*, 37(6), 650–652.
- Collura, T. F. (2001). *Coherence calculation and quadrature filtering*. Oakwood Village, OH: BrainMaster Technologies, Inc.
- Collura, T. F. (2008a). *Multivariate targeting methods for biofeedback [computer software]*. U.S., Canadian, and European Patent Pending.
- Collura, T. F. (2008b). Whole-head normalization using live  $z$  scores for connectivity training. *NeuroConnections*, April 2008, 12–19.
- Collura, T. F., Thatcher, R. W., Smith, M. L., Lambos, W. A., & Stark, C. R. (2008). EEG biofeedback training using live  $z$ -scores and a normative database. In J. Evans, T. Budzynski, H. Budzynski, & D. Arbanel (Eds.), *Quantitative EEG and neurofeedback* (2nd ed.). Amsterdam: Elsevier.
- Crane, R. A. & Soutar, R. (2000). *Mind Fitness training*. Lincoln, NE: Writers Club.
- Fehmi, L. & Collura, T. F. (2007). Effects of electrode placement upon EEG biofeedback Training. *Journal of Neuropathy*, 11(2), 45–63.
- Joffe, D. (1992) *Lexicor NRS-24 BioLex Operator's Manual*. Boulder, CO: Lexicor Corporation.
- McNight, J. T., & Fehmi, L. G. (2001). Attention and neurofeedback synchrony training: Clinical results and their significance. *Journal of Neurotherapy*, 5(1), 45–61.
- Shaw, J. C. (2003). *The brain's alpha rhythms and the mind* (p. 19). New York: Elsevier.
- Smith, M. L. (2008, April). A father finds a solution:  $Z$  score training. *NeuroConnections*.
- Sterman, M. B., & Kaiser, D. A. (2001). Comodulation: A new QEEG analysis metric for assessment of structural and functional disorders of the CNS. *Journal of Neurotherapy*, 4, 73–84.
- Thatcher, R. W., Krause, P. J., & Hrybyk, M. (1986) Cortico-cortical associations and EEG coherence: A two-compartmental model. *Electroencephalography and Clinical Neurophysiology*, 64, 123–143.
- Thatcher, R. W. (2007). *Hand calculations of EEG coherence, phase delays and brain connectivity*. St. Petersburg, FL: Applied Neurosciences, Inc.

- Thornton, K. (1999, August). Exploratory investigation into mild brain injury and discriminant analysis with high frequency bands (32–64 Hz). *Brain Injury*, pp. 477–488.
- Thornton, K. (2000). Exploratory analysis: Mild head injury, discriminant analysis with high frequency bands (32–64 Hz) under attentional activation conditions & does time heal?. *Journal of Neurotherapy*, 3, 1–10.
- Walker, J. E., Kozlowski, G. P., & Lawson, R. (2007). A modular activation/coherence approach to evaluating clinical/QEEG correlations. *Journal of Neurotherapy*, 11(1), 2544.
- Walker, J. E., Norman, C. A., & Weber, R. K. (2002). Impact of qEEG-guided coherence training for patients with a mild closed head injury. *Journal of Neurotherapy*, 6(2), 31–43.